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Article

Several attributes of εp – Open Sets in Cluster Topological Space

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ABSTRACT: In this paper, we deals with the general concept of ε- open set in the cluster topological space and also introduce the setε_p-open set in the cluster topological space.Also, we discuss the properties of εp – open sets in the cluster topological space.
Keywords: ε- open set, π-network, cluster and εp – open sets.

INTRODUCTION

Importance of cluster system in topological spaces by R.Thangamariappan and V Renukadevi [7]. In [3], a new class of sets was defined and discussed. More properties and characterization of ε sets are given in [3]. In this paper we define and study a new class of ε_p – open sets in cluster topological spaces.

PRELIMINARIES

Any nonempty system $\varepsilon \subset 2^x - \{\emptyset\}$will be called a cluster system in X. If any nonempty open subset of a nonempty open subset G contains a setfrom ε, then $\varepsilon$is called a π-network in G. For a cluster system $\varepsilon$and a subset A of a space X, we define the set ε (A) of all points x ∈X such that for any neighborhood U of x, the intersection U ∩ A contains a set from ε. A triplet (X,τ, ε) is called a cluster topological space. We consider the cluster system with the property H. we havecl_ε: $2^x \to 2^x$defined by cl_ε (A) =A ∪ ε (A) is a Kuratowski closure operator on 2^x. We will denote by τ $_\varepsilon$the topology generated by cl_ε, called ε -topology, where $\tau$is the original topology on X, that is, τ_ε= $\{A \subset X / cl_\varepsilon(X - A) = X - A\}$. If ε= $2^x - \{\emptyset\}$or ε= $\{x/$ for every x ∈X}, then ε (A) = $cl(A)$. Hence in this case, $cl_\varepsilon = cl(A)$= $cl(A)$ and τ_ε= τ.

Definition: 1.1

A subset A of a cluster topological space (X, τ, ε) is said to be
$\varepsilon -$ open, if A $\subset \varepsilon(A)$
$\varepsilon -$ perfect, if $\varepsilon(A) = A$
A is locally ε scattered if A$\cap\varepsilon(A) = \emptyset$

Lemma: 1.2

(1) ε $(\emptyset)$ = $\emptyset$,
(2) ε (A) is closed,
(3) ε (A) $\subset \bar{A}$,
(4) if A1 ⊂A2, then ε(A1) ⊂ ε(A2),
(5) $\varepsilon$is a π -network in an open set G ≠ $\emptyset$ iff ε (G) = ε $(\bar{G})$ = $\bar{G}$.

Theorem 1.3

Let ((X, τ) be a space with cluster systems$\varepsilon$and $\varepsilon_1$on X, andlet A and B be subsets of X. Then the following hold,
(a) $\varepsilon(\varepsilon(A)) \subseteq \varepsilon(A)$.
(b) *If* $\varepsilon \subset \varepsilon_1$, *then* $E(A) \subseteq \varepsilon_1(A)$.
(c) $\varepsilon(A)$ *is closed,* $\varepsilon(A) \subset cl(A)$ *and if* $A \subset B$, *then* $\varepsilon(A) \subseteq \varepsilon(B)$
(d) *If* $U \in \tau$, *then* $U \cap \varepsilon(A) = U \subseteq \varepsilon(U \cap A)$.
Theorem: 1.4
Let (X, τ, ε) *be a space&*$A,B \subset X$. *If* ε is a cluster system

With the property H, then the following hold,
(a) $\varepsilon(A \cup B) = \varepsilon(A) \cup \varepsilon(B)$.
(b) $\varepsilon(A) - \varepsilon(B) = \varepsilon(A - B) - \varepsilon(B) \subset \varepsilon(A - B)$.

Theorem: 1.5

Let (X,τ,ε) *be a space &*$A \subset X$. If ε is a π-network in X, then $cl(int\{A\}) = \varepsilon(A)$, if every element of ε has nonempty interior.

Theorem:1.6

Let (X,τ,ε) be a cluster topological space, $cl_{\xi}(A) = A \cup \xi(A)$& $A, B \subset X$ .then
1.$cl_{\varepsilon}(\varphi) = \varphi$
2.$A \subseteq cl_{\varepsilon}(A)$
3.$cl_{\varepsilon}(A \cup B) = cl_{\varepsilon}(A) \cup cl_{\varepsilon}(B)$
4.$cl_{\varepsilon}(A) = cl_{\varepsilon}(cl_{\varepsilon}(A))$.

Proof:

By lemma 1.2 we obtain
1. $cl_{\varepsilon}(\varphi) = \varphi \cup \varepsilon(\varphi) = \varphi$
2. $A \subseteq A \cup \varepsilon(A) = cl_{\varepsilon}(A)$
3.$cl_{\varepsilon}(A \cup B) = \varepsilon(A \cup B) \cup (A \cup B)$
$= (\varepsilon(A) \cup \varepsilon(B)) \cup (A \cup B)$
$= (\varepsilon(A) \cup A) \cup (\varepsilon(B) \cup B)$
$= cl_{\varepsilon}(A) \cup cl_{\varepsilon}(B)$
4.$cl_{\varepsilon}(cl_{\varepsilon}(A)) = cl_{\varepsilon}(A \cup \varepsilon(A))$
$= \varepsilon(A \cup \varepsilon(A)) \cup (A \cup \varepsilon(A))$
$= (\varepsilon(A) \cup A) \cup (\varepsilon(A) \cup A)$
$= (A \cup \varepsilon(A)$
$= cl_{\varepsilon}(A)$.

Theorem: 1.7

Let (X,τ,ε) be a ε –topology & A, B $\subset X$. Then
If A$\subset B$, then $cl_{\varepsilon}(A) \subseteq cl_{\varepsilon}(B)$ $cl_{\varepsilon}(A \cup B) \subseteq cl_{\varepsilon}(A) \cap cl_{\varepsilon}(B)$
If U is ε -open, then $U \cap cl_{\varepsilon}(A) \subseteq cl_{\varepsilon}(U \cap A)$

Proof:

1. Since A$\subseteq B$ by thrm 1.3 we have
$cl_{\varepsilon}(A) = A \cup \varepsilon(A) \subseteq B \cup \varepsilon(B) = cl_{\varepsilon}(B)$
2. This is obvious.
3. Since U is ε-open, then
We have $U \cap cl_{\varepsilon}(A) = U \cap (A \cup \varepsilon(A))$
$= (U \cap A) \cup (U \cap \varepsilon(A)$
$\subseteq (U \cap A) \cup \varepsilon(U \cap A)$
$\subseteq cl_{\varepsilon}(U \cap A)$.

2. ε- open sets

Theorem: 2.1

A singleton subset of a space (X,τ) is $\varepsilon - open$ iff it is open.

Proof:

Let $\{x\}$ be a$\varepsilon - open$ subset of X.
Then $\{x\} \subset \varepsilon(A)$
$\{x\} \subset cl(int\{x\})$
Since each singleton subset of any space $\varepsilon - closed$
Then $cl(int\{x\}) = int\,\{x\}$
Thus $\{x\} \subset (int\{x\})$
Hence $\{x\}$ open.

Example: 2.2

Let X $= \{a,b,c\}\tau = \{\emptyset,\{a,b\},X\}\varepsilon = \{\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\},X\}$.Then ε is a cluster on the space(X,τ). Let A=$\{a,c\}, B = \{b,c\}$. Then $\varepsilon(A) = \varepsilon(B)$.Thus A and B are ε open. But A$\cap B$ is not $\varepsilon - open$.

Theorem:2.3

Let (X,τ) be space&ε is π-network in X, if $A \subset X$is $\varepsilon - closed$, then A$\subset int\, cl(A)$

Proof:

Let A be a$\varepsilon - closed$
$\Rightarrow$ X-A is $\varepsilon - open$ $(A \subset \varepsilon(A))$
$\Rightarrow$X-A$\subset$ $\varepsilon(X - A)$

$\Rightarrow cl(int(X-A)) \subset X-A$
$\Rightarrow A \subset X - cl(int(X-A))$
$\Rightarrow A \subset int(cl(A))$.

3. ε_p − open sets

Definition:3.1

A subset A of a cluster topological space (X, τ, ε) *is said to be* ε_p − open if $A \subset Int\left(cl_\varepsilon(A)\right)$.

The complement of such set is called ε_p − closed. The collection of all ε_p − open(res. ε_p –closed)subsets of X will be denoted by ε_p O(X)(resp. ε_p C(X)).

Example: 3.2

$A = \{a, b), X = \{a, b, c\}, \ \tau = \{X, \varphi, \{a\}, \{b\}, \{a\ b\}\}$
$\varepsilon = \{X, \{a\}, \{b\}, \{c\}, \{b, c\}\}$
$cl_\varepsilon(A) = \{a, b\}$
$Int\ (cl_\varepsilon(A)) = \{a, b\}$
Therefore A is ε_p −open.

Example:3.3

Let X = $\{a, b, c, d\}\tau = \{\emptyset, X, \{c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}\}$
$\varepsilon = \{X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{b, c, d\}, \{a, b, c\}\}$
Then A = $\{c\}$ is a ε_p −open but which is not ε *open*.

Theorem: 3.4
Let (X, τ) be space &ε is π-network in X. Then $\varepsilon_p - open\ iff\ G \cap A\ is\ \varepsilon_p - open$.
Proof:
Let A be an $\varepsilon_p - open$ subset of X, let G$\subset X$.
Then $G \cap A \subseteq G \cap int\left(cl_\varepsilon(A)\right)$
$\subseteq Int\left(G \cap cl_\varepsilon(A)\right)$
$\subseteq Int(cl_\varepsilon(G \cap A))$
Hence $G \cap A\ is\ \varepsilon_p - open$.
Proposition: 3.5
Let A be a subset of a cluster topological space (X, τ, ε).Then
$A\ is\ \varepsilon_p - open$iff it is preopen.
$A\ is\ \varepsilon_p - closediff\ it\ is\ semiopen$.
Proof:
If$A\ is\ \varepsilon_p - open$ if and only if $A \subseteq Intcl_\varepsilon(A)$ if and only if $A \subseteq Intcl(A)$ if and only if A is preopen.
If A is $A\ is\ \varepsilon_p - closed$ if and only if $cl_\varepsilon(int(A)) \subset A$ if and only if cl(int(A))$\subseteq A$ if and only if A is semiopen.

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